

PRACTICE SESSION ON SEQUENCES AND SERIES

Exercise 1. Give examples of sequences with the following properties.

- (a) Bounded above but not bounded below.
- (b) Bounded and divergent.
- (c) Bounded and monotonic.

Exercise 2. Let a_n be the sequence defined by the formula $a_n = \frac{n}{2n+3}$ for all $n \geq 1$. Show that a_n is increasing and bounded above.

Exercise 3. Use the definition of convergence of a sequence to prove the following.

- (a) The sequence $(-1)^n$ does not converge to 1.
- (b) The sequence $\frac{1-n}{2n+1}$ converges to $-1/2$.

Exercise 4. Modify the arguments we used for $p = 1$ and $p = 2$ to show that the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots$$

converges if $p > 1$ and diverges if $p < 1$.

Exercise 5. Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^{3n-1} + (-1)^n}{3^{4n}}$. (Hint: Split into two geometric series.)

Exercise 6. Find the sum of the series $\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$. (Hint: Write as a telescoping series.)