PRACTICE SESSION ON DIRECTIONAL DERIVATIVES

Exercise 1. Let $f(x,y) = x^2 - y^2$ and let $\vec{u} = (3/5, 4/5)$.

(a) Use the limit definition of directional derivatives to evaluate $f_{\vec{u}}(1,2)$.

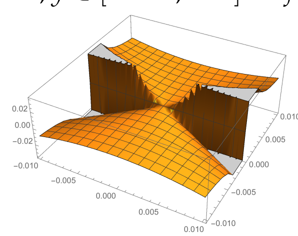
(b) Find $\nabla f(1,2)$.

(c) Use your result in part (b) to evaluate $f_{\vec{u}}(1,2)$ in a different way.

Exercise 2. Let $\vec{u}$ be a unit vector in $\mathbb{R}^2$, and let $\vec{v}$ be the unit vector pointing in the opposite direction of $\vec{u}$. Show that $f_{\vec{v}}(a,b) = -f_{\vec{u}}(a,b)$.

Exercise 3. Let $f(x,y) = \begin{cases} \frac{x^2 y}{x^4+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

(a) Shown below is the graph of $f(x,y)$ for $x, y \in [-0.01, 0.01]$. Is $f(x,y)$ differentiable at $(0,0)$?



(b) Show that $f_{\vec{u}}(0,0)$ exists for every unit vector $\vec{u}$.

Exercise 4. Let $f(x,y) = \sqrt{4 - x^2 - y^2}$.

(a) Draw the graph of $f(x,y)$.

(b) Find the values of $f_x(1,0)$ and $f_y(0,1)$.

(c) The numbers $f_x(1,0)$ and $f_y(0,1)$ represent slopes of tangent lines to certain curves on the graph of $f(x,y)$. Draw these curves and tangent lines on the graph of $f(x,y)$.

(d) Let $\vec{u}$ be the unit vector in the direction of $\vec{i} + \vec{j}$. Find the value of $f_{\vec{u}}(1,1)$ and draw the curve on the graph of $f(x,y)$ and the tangent line to the curve whose slope is $f_{\vec{u}}(1,1)$.

(e) Let $\vec{u}$ be the unit vector in the direction of $\vec{i} - \vec{j}$. Thinking of $f_{\vec{u}}(0,1)$ as the slope of a tangent line, do you expect its value to be positive or negative? Evaluate $f_{\vec{u}}(0,1)$ to confirm your guess.