

PRACTICE SESSION ON TAYLOR POLYNOMIALS

Exercise 1. Find the first, second, and third Taylor polynomials of the function $\sin(\ln x)$ at 1. Then plot the function $\sin(\ln x)$ and the three polynomials in the same window.

Exercise 2. Let $P(x)$ be the n -th Taylor polynomial of $f(x)$ at c . Show that $P^{(k)}(c) = f^{(k)}(c)$ for $k = 0, 1, 2, \dots, n$.

Exercise 3. Use a Taylor polynomial to estimate $\sin(\pi/9)$ with an error smaller than 10^{-8} . How does your estimate compare to your calculator's estimate?

Exercise 4. Use Taylor series to evaluate $\lim_{x \rightarrow 0} \frac{x^2 e^x}{\cos x - 1}$. Check your answer by applying L'Hôpital's Rule or by using a graph.

Exercise 5. Use Taylor series to estimate the value of the definite integral

$$\int_{-0.2}^{0.4} \frac{\cos(x^2)}{(1-x)^3} dx.$$