PRACTICE SESSION ON THE BASIC GEOMETRY OF $\mathbb{R}^n$

Exercise 1. Let $\vec{v} = (2,3)$ and $\vec{w} = (-1,1)$. Use the parallelogram law to draw the following vectors in $\mathbb{R}^2$.

(a) $\vec{v} + 2\vec{w}$

(b) $\vec{w} - \vec{v}$

(c) $\frac{1}{2}\vec{v} + \frac{1}{2}\vec{w}$

Exercise 2. Find a vector in $\mathbb{R}^3$ that has the same direction as $(6,2,-3)$ and has length 4.

Exercise 3. Let $\vec{x}$ and $\vec{y}$ be the following vectors in $\mathbb{R}^4$: $\vec{x} = (1,1,1,1)$ and $\vec{y} = (1,-3,-1,5)$. Find the angle between $\vec{x}$ and $\vec{y}$.

Exercise 4. Let $\vec{x}$ be a nonzero vector in $\mathbb{R}^3$. Let α, β, γ denote the angles between $\vec{x}$ and $\vec{i}, \vec{j}, \vec{k}$, respectively. Determine the value of $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$.

Exercise 5. Let $\vec{v} = (1,-3,1)$ and $\vec{w} = (0,2,4)$, and let θ be the angle between $\vec{v}$ and $\vec{w}$.
Show that $\|v \times w\| = \|v\| \cdot \|w\| \sin\theta$.

Exercise 6. Find a unit vector in $\mathbb{R}^3$ that is ortogonal to both $\vec{i} + \vec{j}$ and $\vec{i} + \vec{k}$. Draw these three vectors in $\mathbb{R}^3$.

Exercise 7. Find a parametric representation of the line in $\mathbb{R}^2$ whose equation is $y = 4x - 7$.

Exercise 8. Find a parametric representation of the line in $\mathbb{R}^3$ that passes through the points $(1,2,4)$ and $(0,-2,3)$.

Exercise 9. Find the equation of the plane in $\mathbb{R}^3$ that contains the origin and the points $(3,-2,1)$ and $(1,1,1)$.

Exercise 10. The intersection of the planes in $\mathbb{R}^3$ with equations $x + y + z = 1$ and $x - 2y + 3z = 1$ is a line. Find a parametric representation of that line.